(1) $F(x,y) = (x^2+y,\ x+y^2)$ $\quad G(u,v) = (\sin(uv),\ \cos(uv))$

$$J_F = \begin{pmatrix} 2x & 1 \\ 1 & 2y \end{pmatrix} \qquad J_G = \begin{pmatrix} v\cdot\cos(uv), & u\cdot\cos(uv) \\ -v\cdot\sin(uv), & -v\cos(uv) \end{pmatrix}$$

$$G\circ F(x,y) = \left(\sin(x^2y^2+x^3+y^3+xy),\ \cos(x^2y^2+x^3+y^3+xy)\right)$$

• Přímo

$$J_{G\circ F} = \begin{pmatrix} (2xy^2+3x^2+y)\cos(x^2y^2+x^3+y^3+xy), & (2yx^2+3y^2+x)\cos(x^2y^2+x^3+y^3+xy) \\ -(2xy^2+3x^2+y)\sin(x^2y^2+x^3+y^3+xy), & -(2yx^2+3y^2+x)\sin(x^2y^2+x^3+y^3+xy) \end{pmatrix}$$

• Pomocí věty o derivaci složeného zobrazení

$$J_{G\circ F} = (J_G \circ F)\cdot J_F = \begin{pmatrix} (x+y^2)\cos(x^2y^2+x^3+y^3+xy), & (x^2+y)\cos(x^2y^2+x^3+y^3+xy) \\ -(x+y^2)\sin(x^2y^2+x^3+y^3+xy), & -(x^2+y)\sin(x^2y^2+x^3+y^3+xy) \end{pmatrix}\cdot\begin{pmatrix} 2x & 1 \\ 1 & 2y \end{pmatrix}$$

$$= \begin{pmatrix} (2x^2+2xy^2+x^2+y)\cos(x^2y^2+x^3+y^3+xy), & (x+y^2+2x^2y+2y^2)\cos(x^2y^2+x^3+y^3+xy) \\ -(\underbrace{2x^2+2xy^2+x^2+y}_{3x^2+2xy^2+y})\sin(x^2y^2+x^3+y^3+xy), & -(\underbrace{x+y^2+2x^2y+2y^2}_{x+2x^2y+3y^2})\sin(x^2y^2+x^3+y^3+xy) \end{pmatrix}$$

(2)

$$f(x,y)=\begin{cases} e^{-\frac{1}{2x^2+3y^2-4xy}} & (x,y)\neq(0,0) \\ 0 & (x,y)=(0,0)\end{cases}$$

$f\in \mathcal{C}^\infty(\mathbb{R}^2\setminus\{(0,0)\})$ tedy má tot. diferenciál na $\mathbb{R}^2\setminus\{(0,0)\}$

$$f(x,0)=\begin{cases} e^{-\frac{1}{2x^2}} & x\neq 0 \\ 0 & x=0\end{cases} \to \frac{\partial f}{\partial x}(0,0)=\lim_{x\to 0}\left(e^{-\frac{1}{2x^2}}\right)'=\lim_{x\to 0}\frac{1}{x^3}e^{-\frac{1}{2x^2}}=0$$

$$f(0,y)=\begin{cases} e^{-\frac{1}{3y^2}} & y\neq 0 \\ 0 & y=0\end{cases} \to \frac{\partial f}{\partial y}(0,0)=\lim_{y\to 0}\left(e^{-\frac{1}{3y^2}}\right)'=\lim_{y\to 0}\frac{2}{3}\cdot\frac{1}{y^3}\cdot e^{-\frac{1}{3y^2}}=0$$

Pokud $f'(0,0)$ existuje, potom $f'(0,0)=0$

Počítáme limitu $$\lim_{(x,y)\to(0,0)}\frac{f(x,y)-f(0,0)-0}{\sqrt{x^2+y^2}}=\lim_{(x,y)\to(0,0)}\underbrace{\frac{e^{-\frac{1}{2x^2+3y^2-4xy}}}{\sqrt{x^2+y^2}}}_{g(x,y)}$$

$$g(r\cos\varphi,r\sin\varphi)=\frac{e^{-\frac{1}{2r^2\cos^2\varphi+3r^2\sin^2\varphi-4\sin\varphi\cos\varphi}}}{r^2}\leq\frac{e^{-\frac{1}{9r^2}}}{r^2}\to 0 \quad r\to 0+$$

$$=r^2(\underbrace{\sin^2\varphi+2(\sin\varphi-\cos\varphi)^2}_{u(\varphi)})$$

$u(\varphi)>0$ a $u(\varphi)\leq 9$

Tedy f má v $(0,0)$ tot. diferenciál.